1 Single-Phase a.c. Circuits

This chapter deals with the solution of parallel circuits, and the effects of both series and parallel resonance. Also included are the technique of power factor correction, and the concepts of tuned circuits and filters.

On completion of the chapter, you should be able to:

1 Determine the current flows, p.d.s and power dissipation in parallel a.c. circuit configurations.
2 State the conditions for resonance in both series and parallel circuits, and apply the formulae to obtain the resonant frequency in each case.
3 Define and calculate circuit Q-factor.
4 Appreciate the need for power factor correction, and carry out the relevant calculations.
5 Understand the effect of turned (resonant) circuits on circuit selectivity and bandwidth, and their application to filter circuits.

1.1 Summary of Series a.c. Circuits

The solution of parallel a.c. circuits is a simple extension of the methods used for series circuits. It is therefore sensible, at this stage, to summarise what has been learned about series circuits.

1 In a series circuit, the current is common to all of the series elements. For this reason the current is chosen as the reference phasor.
2 In a series circuit, the *phasor* sum of the p.d.s equals the total applied voltage.
3 Reactance is the opposition offered, to the flow of a.c., by a pure inductor or capacitor. Reactance is measured in ohms, and the symbols are X_L and X_C.
4 Reactance depends upon the frequency of the supply, such that:

$$X_L = 2\pi f_L \text{ ohm}$$

$$\text{and } X_C = \frac{1}{2\pi fC} \text{ ohm}$$

5 Impedance is the overall opposition offered to the flow of a.c., in a circuit that contains both resistance and reactance. Impedance has the symbol Z, and is measured in ohms.

6 $Z = \dfrac{V}{I} = \sqrt{R^2 + (X_L - X_C)^2}$ ohm

7 Power is dissipated *only* by a resistive component. The power may be calculated from either:

$P = I^2R$ or $P = VI\cos\phi$ watt

where ϕ is the circuit phase angle, and cos ϕ is the circuit power factor.

8 For a pure resistor,

$\phi = 0°$ $\quad V_R$ in phase with I

For a pure inductor,

$\phi = +90°$ $\quad V_L$ leads I by 90°

For a pure capacitor,

$\phi = -90°$ $\quad V_C$ lags I by 90°

the above being summed up by the 'keyword' CIVIL. Typical circuit and phasor diagrams are shown in Figs. 1.1 and 1.2.

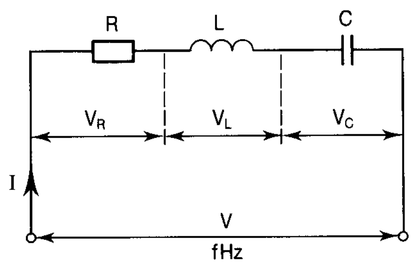

Fig. 1.1

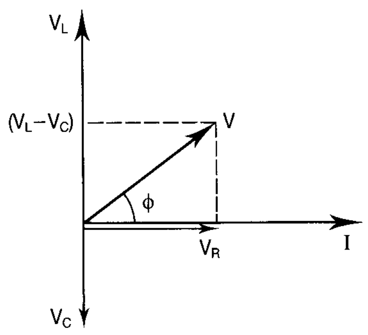

Fig. 1.2

1.2 The *R-C* Parallel Circuit

Consider a pure resistor and a pure capacitor, connected in parallel across an a.c. supply, as shown in Fig. 1.3. Since both components are connected directly to the same supply terminals, then the *supply voltage*, V, is common to them both. On the other hand, each component will draw a current dependent upon its opposition offered.

i.e. $I_1 = \dfrac{V}{R}$ amp, and $I_2 = \dfrac{V}{X_C}$ amp

However, both I_1 and I_2 are supplied from the a.c. source. Thus the total current drawn from the supply, I is the PHASOR SUM of I_1 and I_2. The resulting phasor diagram, as shown in Fig. 1.4, therefore uses the applied voltage V as the reference phasor. From this diagram, it may be seen that

$$I = \sqrt{I_1^2 + I_2^2} \text{ amp}$$

$$\text{and } \cos\phi = \frac{I_1}{I} \quad \text{or } \tan\phi = \frac{I_2}{I_1}$$

Note: In a parallel circuit, the equation $Z = \sqrt{R^2 + X_C^2}$ CAN NOT BE USED to obtain the circuit impedance. The impedance can only be obtained from $Z = V/I$ ohm. Also note the fact that since the current through the capacitor (I_2) *leads* the voltage V, then a capacitor in a parallel circuit results in a leading phase angle, ϕ. This may be confirmed by considering the word CIVIL.

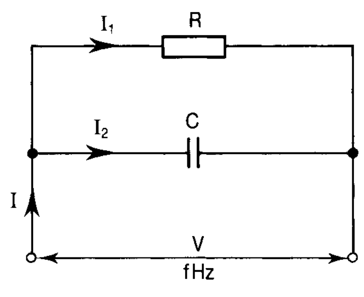

Fig. 1.3

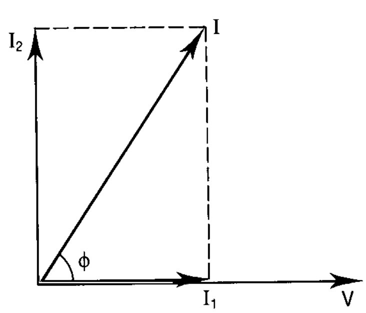

Fig. 1.4

Worked Example 1.1

Question

A 2.2 μF capacitor is connected in parallel with a 1.5 kΩ resistor across a 100 V, 50 Hz supply. Sketch the circuit and phasor diagrams, and calculate (a) the current flow through each component, (b) the current drawn from the supply, (c) the circuit phase angle, and (d) the power dissipated.

Answer

$C = 2.2 \times 10^{-6}$ F; $R = 1500\,\Omega$; $V = 100$ V; $f = 50$ Hz

(a) $$I_1 = \frac{V}{R}\text{ amp} = \frac{100}{1500}$$

so $I_1 = 66.7$ mA **Ans**

$$X_C = \frac{1}{2\pi fC}\text{ ohm} = \frac{1}{2 \times \pi \times 50 \times 2.2 \times 10^{-6}}$$

so $X_C = 1446.9\,\Omega$

$$I_2 = \frac{V}{X_C}\text{ amp} = \frac{100}{1446.9}$$

so $I_2 = 69.1$ mA **Ans**

(b) $$I = \sqrt{I_1^2 + I_2^2}\text{ amp} = \sqrt{66.7^2 + 69.1^2}\text{ mA}$$

hence $I = 96$ mA **Ans**

(c) $$\phi = \cos^{-1}\frac{I_1}{I} = \cos^{-1}\frac{66.7}{96}$$

so $\phi = 46.03^\circ$ leading **Ans**

(d) $$P = VI\cos\phi\text{ watt} = 100 \times 96 \times 10^{-3} \times 0.6943$$

so $P = 6.67$ W **Ans**

Alternatively, $P = I_1^2 R\text{ watt} = (66.7 \times 10^{-3})^2 \times 1500$

so $P = 6.67$ W **Ans**

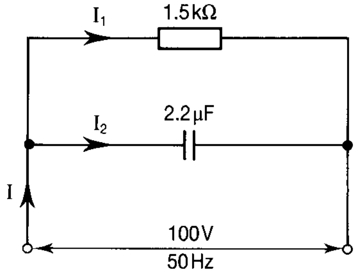

Fig. 1.5

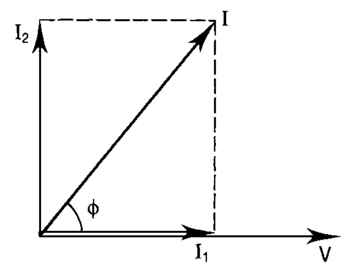

Fig. 1.6

Worked Example 1.2

Question

For the circuit shown in Fig. 1.7, determine (a) the supply voltage, (b) the supply frequency, and (c) the current drawn from the supply.

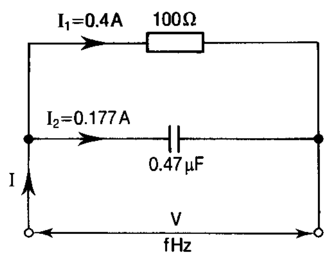

Fig. 1.7

Answer

(a) Since the supply is connected directly across the resistor, then the p.d. across $R = V$.

therefore, $V = I_1 R$ volt $= 0.4 \times 100$

so, $V = 40\,\text{V}$ **Ans**

(b) $X_C = \dfrac{V}{I_2}$ ohm $= \dfrac{40}{0.177}$

therefore, $X_C = 225.99\,\Omega$

and, since $X_C = \dfrac{1}{2\pi f C}$ ohm

then, $f = \dfrac{1}{2\pi C X_C}$ hertz

therefore, $X_C = 225.99\,\Omega$

and, since $X_C = \dfrac{1}{2\pi f C}$ ohm

then, $f = \dfrac{1}{2\pi C X_C}$ hertz

so, $f = \dfrac{1}{2 \times \pi \times 225.99 \times 0.47 \times 10^{-6}}$

hence, $f = 1498\,\text{Hz}$ **Ans**

(c) $I = \sqrt{I_1^2 + I_2^2}$ amp $= \sqrt{0.4^2 + 0.177^2}$

thus, $I = 0.437\,\text{A}$ **Ans**

1.3 The *R-L* Parallel Circuit

When a pure inductor is connected in parallel with a resistor, the techniques used to analyse the circuit are the same as for the *C-R* circuit. The only difference will be that the circuit will have a lagging phase angle. The following example illustrates this.

Worked Example 1.3

Question

The circuit of Fig. 1.8 shows a pure inductor connected in parallel with a 200 Ω resistor, across a 50 V, 1 kHz supply. If the current drawn from the supply is 1.03 A at a power factor of 0.2427, determine (a) the current in each branch, (b) the power dissipated, and (c) the value of the inductance.

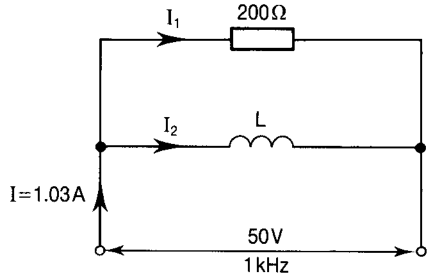

Fig. 1.8

Answer

$V = 50\,\text{V};\; f = 10^3\,\text{Hz};\; I = 1.03\,\text{A};\; \cos\phi = 0.2427$

The corresponding phasor diagram is shown in Fig. 1.9.

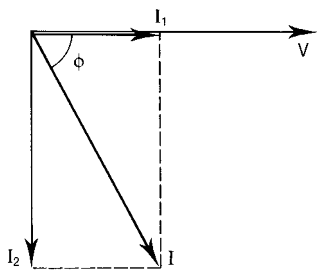

Fig. 1.9

(a) $\phi = \cos^{-1} 0.2427 = 75.95^\circ$

$$I_1 = \frac{V}{R}\text{ amp} = \frac{50}{200}$$

hence, $I_1 = 0.25\,\text{A}$ **Ans**

From the phasor diagram, it may be seen that:

$$I_2 = \sqrt{I^2 - I_1^2}\text{ amp} = \sqrt{1.03^2 - 0.25^2}$$

therefore, $I_2 = 1\,\text{A}$ **Ans**

(b) $P = VI\cos\phi\text{ watt} = 50 \times 1.03 \times 0.2427$

so $P = 12.5\,\text{W}$ **Ans**

(c) $X_L = \frac{V}{I_2}\text{ ohm} = \frac{50}{1}$

so, $X_L = 50\,\Omega$

but, $X_L = 2\pi fL\text{ ohm}$,

so, $L = \frac{X_L}{2\pi f}\text{ henry} = \frac{50}{2 \times \pi \times 10^3}$

hence, $L = 7.96\,\text{mH}$ **Ans**

1.4 *R-L-C* Parallel Circuit

This circuit is a simple extension of the previous two considered. The only differences are that, (a) there are three branch currents, and (b) the circuit phase angle may be either leading or lagging. The latter depends upon the relative values of the currents through inductor and capacitor.

Worked Example 1.4

Question

A $75\,\Omega$ resistor, an 80 mH inductor, and a $40\,\mu\text{F}$ capacitor are connected in parallel across an 80 V, 100 Hz supply. Sketch the circuit and phasor diagrams, and calculate (a) the three branch currents, (b) the current drawn from the supply, and (c) the circuit phase angle and power factor.

Answer

$R = 75\,\Omega$; $L = 80 \times 10^{-3}\,\text{H}$; $C = 44 \times 10^{-6}\,\text{F}$; $V = 80\,\text{V}$; $f = 100\,\text{Hz}$

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Worked Example 1.4 (*Continued*)

The circuit and phasor diagrams are shown in Figs. 1.10 and 1.11 respectively.

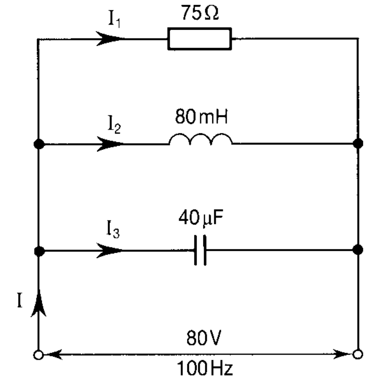

Fig. 1.10

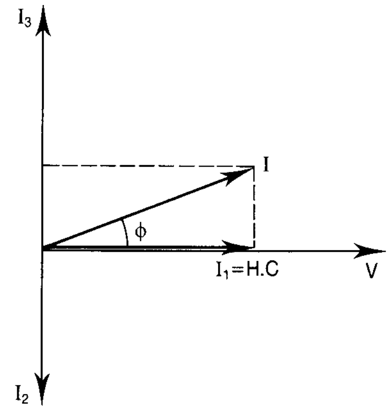

Fig. 1.11

(a) $I_1 = \dfrac{V}{R}\,\text{amp} = \dfrac{80}{75}$

so, $I_1 = 1.067\,\text{A}$ **Ans**

$X_L = 2\pi fL\,\text{ohm} = 2 \times \pi \times 100 \times 80 \times 10^{-3}$

$X_L = 50.265\ \Omega$

$I_2 = \dfrac{V}{X_L}\,\text{amp} = \dfrac{80}{50.265}$

and, $I_2 = 1.592\,\text{A}$ **Ans**

$$X_C = \frac{1}{2\pi fC}\ \text{ohm} = \frac{1}{2 \times \pi \times 100 \times 40 \times 10^{-6}}$$

so $X_C = 39.8\,\Omega$

$$I_3 = \frac{V}{X_C}\ \text{amp} = \frac{80}{39.8}$$

hence, $I_3 = 2.01$ A **Ans**

(b) From the phasor diagram:

Horizontal component, H.C. $= I_1 = 1.067$ A

Vertical component, V.C. $= I_3 - I_2 = 2.01 - 1.592$

so, V.C. $= 0.418$ A

$$I = \sqrt{\text{V.C.}^2 + \text{H.C.}^2} = \sqrt{0.418^2 + 1.067^2}$$

therefore, $I = 1.146$ A **Ans**

$$\phi = \tan^{-1}\frac{\text{V.C.}}{\text{H.C.}} = \tan^{-1}\frac{0.418}{1.067}$$

hence, $\phi = 21.4^\circ$ leading **Ans**

power factor, $\cos\phi = 0.931$ leading **Ans**

1.5 Practical Components in Parallel

Thus far, we have considered that the circuit components are perfect: i.e. pure resistor, pure inductor and pure capacitor. In practice, unless the resistor is of the wirewound type, it is normally assumed to be purely resistive. Similarly, provided that the frequency is not very high, or the capacitor is not an electrolytic type, then the capacitor may also be considered as perfect. The inductor, on the other hand, may rarely be considered as pure inductance. Thus, the resistance of the inductor should be taken into account. The inductor coil therefore has an impedance, Z_{coil} ohm.

A circuit commonly met in practice consists of a 'practical' inductor, connected in parallel with a 'perfect' capacitor. Such a circuit is illustrated in Fig. 1.12, which could represent the coil of an a.c. motor, with a parallel-connected capacitor.

$$\text{The current through the coil, } I_1 = \frac{V}{Z_1} \text{ amp}$$
$$\text{where impedence of coil} = Z_1 = \sqrt{R^2 + X_L^2} \text{ ohm}$$
$$\text{and } \phi_1 = \cos^{-1}\frac{R}{Z_1} \text{ (lagging } V\text{)}$$

Current through the capacitor, $I_2 = V/X_C$ amp and this current will lead the applied voltage by 90°.

The phasor diagram is shown in Fig. 1.13. From this diagram it may be seen that the capacitor current forms a vertical component only. The inductor coil current has both horizontal and vertical components, $I_1 \cos \phi_1$ and $I_1 \sin \phi_1$ respectively. The total vertical component (V.C.) is therefore the difference between I_2 and $I_1 \sin \phi_1$. A supplementary phasor diagram is shown in Fig. 1.14, from which the circuit current and phase angle can be determined.

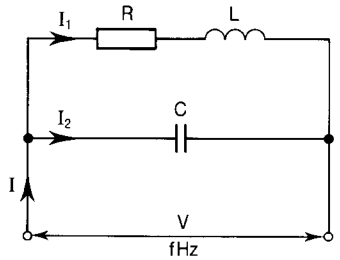

Fig. 1.12

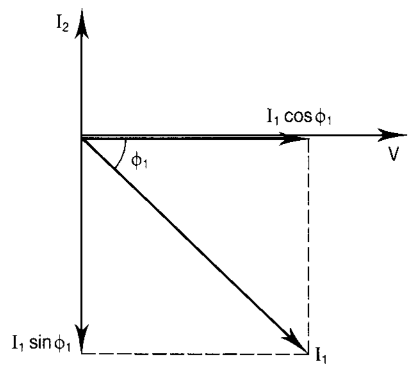

Fig. 1.13

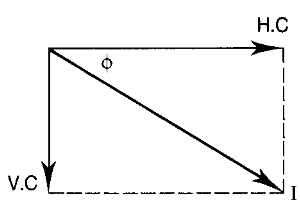

Fig. 1.14